National Taiwan University of Science and Technology

Graduate Institute of Intelligent Manufacturing Technology

Ph.D. Qualifying Examination

Subject	Robotics Engineering and Automation Technology		
Date	2025 / 05 / 06	Time	09:00 (Start) to 12:00 (End) (Total 180 Minutes)

Instructions to Candidates:

1. This exam paper consists of **SIX** pages (including this page). Please verify the number of pages.
2. Do not write any text or symbols unrelated to the answers on the exam paper.
3. Answers should be written on the answer sheet, with clear indication of the question number.
4. Upon the announcement of the exam end time, please stop writing immediately and submit both the exam paper and the answer sheet.
5. Violation of examination rules will be dealt with according to university regulations.
6. Reference materials not allowed in this exam.

Full Name: ______________________________ Student ID: ____________

Question 1. [25%]

The Denavit-Hartenberg (D-H) method is a standard approach used in kinematics analysis of robot manipulators. The process for assigning D-H frames to robot links should follow these rules:

1. The z-axis must be the axis of rotation for a revolute joint, or the direction of motion for a prismatic joint.
2. The x-axis must be perpendicular both to its own z-axis, and the z-axis of the frame before it.
3. All frames must follow the right-hand rule.

As shown in Figure 2-1, the four D-H parameters are defined as:

- a_i = offset distance between z_{i-1}-axis and z_i-axis along x_i-axis
- α_i = angle between z_{i-1}-axis and z_i-axis about x_i-axis
- d_i = offset distance between x_{i-1}-axis and x_i-axis along z_{i-1}-axis
- θ_i = angle between x_{i-1}-axis and x_i-axis about z_{i-1}-axis

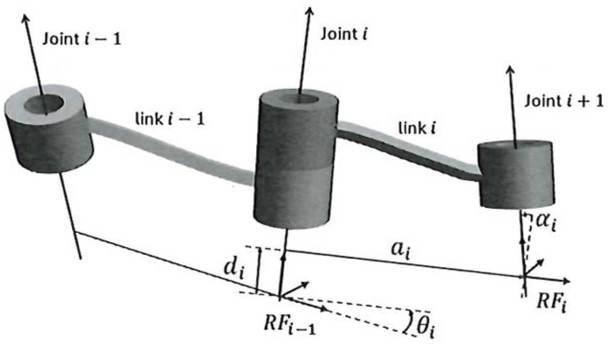

Figure 2-1 Denavit-Hartenberg Parameters

Figure 2-2 shows a SCARA robot in its zero configuration. The ground frame is $\{x_0, y_o, z_0\}$ and the end-effector frame is $\{x_4, y_4, z_4\}$. Based on the figure, answer the following:

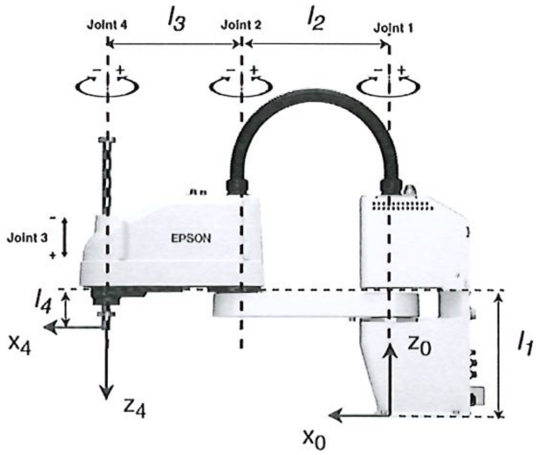

Figure 2-2 SCARA Robot

(a) Draw a schematic representation of the robot. [5%]

(b) Complete the D-H parameter table for the SCARA robot. [8%]

Joint i	a_i	α_i	d_i	θ_i
1				
2				
3				
4				

(c) Write the individual transformation matrices from frame $i-1$ to i for $(i = 1 \sim 4)$. [4%]

(d) Compute the forward kinematics using the parameters givne in Table 2-1. [8%]

Table 2-1

Links	l_1	l_2	l_3	l_4
Link length (mm)	80	100	100	30
Joints	q_1	q_2	q_3	q_4
Joint value	0	$-\frac{\pi}{2}$	35 mm	π

Question 2

(a) The matrix J below is the Jacobian of a spatial 6-DOF robot. Determine whether the robot is in a singular configuration, and breify explain. [5%]

$$J = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \\ 0 & -5 & 6 & -\frac{4}{5} & -\frac{4}{5}+\frac{5}{2}\sqrt{3} & -\frac{4}{5}+\frac{5}{2}\sqrt{3} \\ 0 & 0 & 0 & 0 & \frac{-5}{2} & 6 \end{bmatrix}$$

Figure 3-1 shows a planar 3-DOF manipulator.

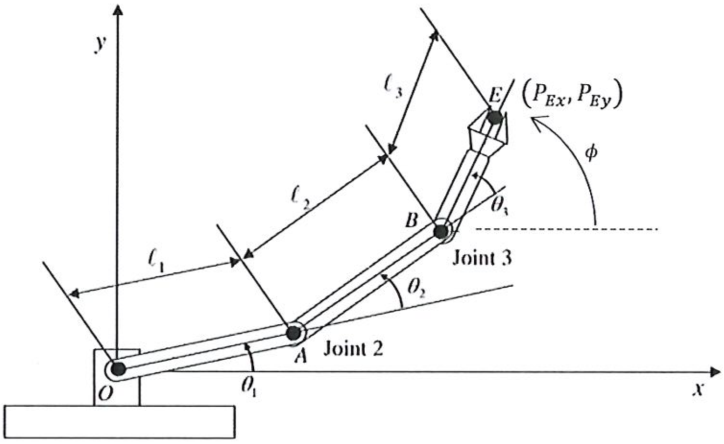

Figure 3-1 A Planar 3-DOF Manipulator

(b) Derive the analytical forward kinematics of the manipulator. [5%]
(Given: $\theta_1, \theta_2, \theta_3, l_1, l_2, l_3$; Find: P_{Ex}, P_{Ey}, ϕ)

(c) Derive the analytical inverse kinematics solutions of the manipulator. [10%]
(Given: $P_{Ex}, P_{Ey}, \phi, l_1, l_2, l_3$; Find: $\theta_1, \theta_2, \theta_3$)

(d) Derive the Jacobian matrix of the manipulator using the link lengthes and joint values in Table 3-1. [5%]

Table 3-1

Links	l_1	l_2	l_3
Link length (mm)	60	60	30
Joints	θ_1	θ_2	θ_3
Joint values	$\frac{\pi}{2}$	0	$-\frac{\pi}{4}$

Formulae

Trigonometric identities

$$\sin(\theta_1 \pm \theta_2) = \sin(\theta_1)\cos(\theta_2) \pm \cos(\theta_1)\sin(\theta_2)$$

$$\cos(\theta_1 \pm \theta_2) = cos(\theta_1)\cos(\theta_2) \mp \sin(\theta_1)\sin(\theta_2)$$

Standard rotation matrices

$$\boldsymbol{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}, \quad \boldsymbol{R}_y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}, \quad \boldsymbol{R}_z(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix with four DH parameters

$$^{i-1}A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 3. [25%]

There is a 3-dimensional point [1,2,3] in the X-Y-Z coordinate. Compute the position of the point after performing the following transformations in a row: (a) rotation about X axis by 30 degrees, (b) rotation about Z axis by -45 degrees, (c) translation along the Y axis by 10 units.

Question 4. [25%]

Below please find a 6R robot arm with the information of some geometrical dimensions and axis directions. Please provide the complete DH table for this robot arm. Note: use θ to represent the joint angle; use d to represent the link offset; use a to represent the link length; use α to represent the link twist.